NOVEMBER

NCTM NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Each day Santos makes an open-faced sandwich using only one slice of bread. He uses 1 kind of meat or 1 kind of cheese or 1 kind of each. If he chooses from 4 kinds of bread, 5 kinds of meat, and 3 kinds of cheese, how many different sandwiches can he make?

1

How many integers between 1000 and 9999 have distinct digits?

2

To express 2010 in base 2, we use exactly 8 ones and 3 zeros. Find the next two integers in base 2 that use the same number of ones and zeros.

3

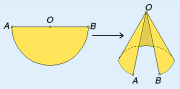
By what percent above cost must a merchant mark up goods in order to offer a 20% discount and still make a 20% profit?

4

Ten boys and girls write their names on slips of paper—one name per slip—to enter a prize drawing. Two of the names are drawn at random without replacement. If the probability that both winners are boys is 1/15, how many boys are in the group?

5

A paper semicircle has diameter $AB = 2$. The two halves of that edge ($\overline{OA}$ and $\overline{OB}$) are joined to make a cone. Find the measure of the cone's opening angle (the vertex angle made by a cross section through the apex and the center of the base).



6

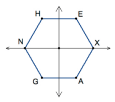
All diagonals have been constructed in a convex pentagon. How many triangles of any size are created by the sides and the diagonals?

7

Select one vertex, V, of a cube. How many paths begin at V, traverse exactly 3 edges of the cube, and end at the vertex furthest from V?

8

Regular hexagon $HEXAGN$ has side length 5 and center at the origin. Sides HE and AG are parallel to the x-axis. Translation T maps $HEXAGN$ to $H'E'X'A'G'N'$ such that N' and H are the same point. What is the sum of the x-coordinates of points H', E', X', A', G', and N'?



9

To raise $5,999,400 in taxes, a town charged every taxpayer the same amount. After the tax bill was calculated but before the taxes were due, 12 taxpayers moved away. Revised tax bills required each remaining taxpayer to pay an additional $200. How many taxpayers remained in the town?

10

Find all integers n for which

$$n^2 + 6n - 27$$

is a prime number.

11

What is the smallest composite number generated by

$$p^2 - p - 1$$

where p is prime?

12

Define a recursive sequence a_n as follows:

$$a_1 = 7,$$
$$a_2 = 3,$$
and, for $n \geq 3$,
$$a_n = a_{n-1} - a_{n-2}.$$

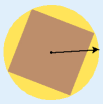
Find the value of a_{2010}.

13

If Jack and Jill each randomly choose a whole number from 1 to 1000 inclusive, what is the probability that Jill's number is larger than Jack's?

14

The cardboard base of a spinner has a square inscribed in a circle with radius 3 in. The probability that the 3-in. pointer will land on brown is essentially 0. What length should the pointer be so that $P(\text{yellow}) = P(\text{brown})$?



15

Commuter trains leave from Middletown for Centerville every hour on the hour. The trip takes 3 hours. Each train waits at the Centerville train depot for a half hour and then returns to Middletown. If the return trip also takes 3 hours, how many trains will each train pass (going in the opposite direction) during the round trip?

16

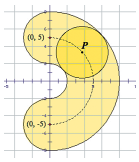
Nine students—3 from Mr. Boe's class, 3 from Mrs. Coe's class, and 3 from Ms. Doe's class—have bought a block of 9 seats in a row for their school's homecoming game. If the 9 seats are randomly assigned, what is the probability that students from each class will sit together in a block of 3 consecutive seats?

17

Imagine the product of 20^{50} and 50^{20} written as an integer in standard form. How many zeros will be found at the end of this number?

18

The center of circle P travels along a semicircular path from $(0, 5)$ to $(0, -5)$. If the circle sweeps out an area of 39π units^2, what is the radius of circle P?



19

Suppose that the diameter of a circle has length d. The length of a chord is c, and the length of the arc cut off by the chord is s. Express $\sin(s/d)$ in terms of c and d.

20

Mental math—neither calculator nor pencil needed! The sum of the first 50 positive odd integers is 50^2. Find the sum of the first 50 positive even integers.

21

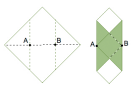
Ryan invests M dollars in the stock market. After his investment increases by 10%, he takes out $150 and sets it aside. The remaining investment then decreases by 10%. If the sum of the invested money and the $150 set aside are 4% larger than the original investment M, by what dollar amount has M increased?

22

Triangle ABC is equilateral. $\overrightarrow{CG}$ and $\overrightarrow{CB}$ are opposite rays, and D is a point on $\overrightarrow{CG}$ such that $CB = 2CD$. F is a point between A and B such that $\overline{FD} \perp \overline{AB}$. Let E be the intersection of $\overline{FD}$ and $\overline{AC}$. If $AF = 6$, find the area of quadrilateral $FBCE$.

23

A paper square has diagonal length 6 in. Two folds are made along lines perpendicular to the diagonal and through trisection points A and B. By what percentage has the visible area of the paper decreased?



24

Find the area of the region bounded by the graph of $y = ||x| - 5|$ and the graph of $y = 5$.

25

Find the three-digit number that divides the expression

$$n^5 - 5n^3 + 4n$$

evenly for all integers n.

26

Given this system of equations

$$y = -x + 6$$
$$y = x/3 + c$$

with the solution lying in quadrant I, find all possible values of c.

27

ABC is a right triangle with legs $BA = 8$ and $BC = 6$. Point D is between B and C such that $BD = 5$. Point E lies on $\overline{BA}$ extended such that $\overline{BE} = 12$. Point F is the intersection of $\overline{AC}$ and $\overline{DE}$. Find the distance from D to F.

28

Trapezoid $TRAP$ has vertices $T(1, 1)$, $R(2, 4)$, $A(5, 4)$, and $P(10, 1)$. $TRAP$ is dilated about the origin with scale factor $-1/2$. If Z is the midpoint of the median of $TRAP$, what are the coordinates of Z', the image of Z under the dilation?

29

Given the quadratic polynomial $ax^2 + bx + c$, find a new polynomial with coefficients expressed in terms of a, b, and c such that the product and the sum of its zeros will be the sum and the product, respectively, of the zeros of the original polynomial.

30